5 Rossby Waves

Suggested Literature:

1. Hoskins, B.J. and Karoly D.J., 1981: 'The Steady Linear Response of a Spherical Atmosphere to Thermal and Orographic Forcing', *J. Climate*, **38**, 1179-1196

5.1 Free Barotropic Rossby Waves

The dispersion relation for free barotropic Rossby waves can be derived by linearizing the barotropic vorticity equation in the form (119). This equation states that the absolute (geostrophic) vorticity is conserved following the horizontal (geostrophic) motion. As usual, we assume that the fields can be expressed as small perturbations from a basic state $\psi = \overline{\psi} + \psi'$. We linearize using a basic state that has only flow in zonal direction $\overline{\psi} = -\overline{u}y + const$. This mean state fulfills Eq. (119). With this mean state $\nabla^2\psi = \nabla^2\psi'$. Thus, by linearizing, in the first term the total derivative operator can be replaced by the mean operator and it follows

$$\left(\frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x}\right)\nabla^2\psi' + \beta\frac{\partial\psi'}{\partial x} = 0 \quad . \tag{139}$$

As usual, we seek for solutions of the type

$$\psi' = Ae^{i(kx+ly-\nu t)} \quad . \tag{140}$$

Inserting (140) into (139) yields the dispersion relation

$$(-\nu + k\overline{u})(-k^2 - l^2) + k\beta = 0 \quad , \tag{141}$$

which we can solve immediately for ν

$$\nu = \overline{u}k - \beta k/K^2 \quad , \tag{142}$$

where $K^2 \equiv k^2 + l^2$ is the total horizontal wave number squared. Recalling that $c_x = \nu/k$, we find that the zonal phase speed relative to the mean wind is

$$c_x - \overline{u} = -\beta/K^2 \quad . \tag{143}$$

Thus, the Rossby wave zonal phase propagation is always westward relative to the mean zonal flow. Furthermore, the Rossby wave phase speed depends inversely on the square of the horizontal wave number. Therefore, Rossby waves are dispersive waves whose phases speeds increase rapidly with increasing wavelength. This result is consistent with the fact that the advection of planetary vorticity, which tends to make the disturbances *retrogress*, increasingly dominates over relative vorticity advection as the wavelength of a disturbance increases. Equation (143) provides a quantitative measure of this effect in cases where the disturbance is small enough in amplitude.

From Eq. (143) we may calculate the stationary free Rossby wave wavelength

$$K^2 = \beta/\overline{u} \equiv K_s^2 \quad . \tag{144}$$

This means that stationary free Rossby waves only exist if there is a positive mean flow $\overline{u}$. This condition is important for Rossby waves that may be generated by tropical convection.

The group velocity of Rossby waves may be calculated as (exercise!):

$$c_{gx} \equiv \frac{\partial \nu}{\partial k} = \overline{u} + \beta \frac{k^2 - l^2}{K^4} \tag{145}$$

$$c_{gy} \equiv \frac{\partial \nu}{\partial l} = 2\frac{\beta k l}{K^4} \quad . \tag{146}$$

Therefore, the energy propagation of stationary Rossby waves is always eastward (Fig. 13; exercise!).

These waves can also be derived from the original, compressible equations, but the analysis is much more complicated. There are some minor modifications in the phase velocities if the full equations are considered, but the main results remain valid.

5.2 Forced Topographic Rossby waves

Forced stationary Rossby waves are of primary importance for understanding the planetary-scale circulation pattern. Such modes may be forced by longitudinal dependent latent heating, or by flow over topography. Of particular importance for the Northern Hemisphere extratropical circulation are stationary Rossby modes forced by flow over the Rockies and the Himalayas.

As the simplest possible dynamical model of topographic Rossby waves we use the barotropic vorticity equation for a homogeneous fluid of variable depth (e.g. Eqs. 112 or 115). We assume that the upper boundary is at fixed height H and the lower boundary is at the variable height $h_T(x, y)$. We also use the quasi-geostrophic scaling $|\xi| \ll f_0$. Then, from 112 and 115 we have

$$H \frac{d_h(\xi + f)}{dt} = -f_0 \frac{dh_T}{dt} \quad , \tag{147}$$

where is has been also assumed that $h \equiv H - h_T \approx H$ on the left side (i.e. the mountain height is much smaller than the troposphere height). After linearizing (as we did to derive Eq. 139)

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)\nabla^2\psi' + \beta\frac{\partial \psi'}{\partial x} = -\frac{f_0}{H}\bar{u}\frac{\partial h_T}{\partial x} \quad . \tag{148}$$

Lets consider the solutions of Eq. (148) for the special case of a sinusoidal lower boundary. We specify the topography to have the from

$$h_T(x, y) = h_0 \sin(kx + \phi) \cos ly \quad , \tag{149}$$

where ϕ is an arbitrary phase (therefore equivalent to $A\cos kx + B\sin kx$). If we insert the streamfunction perturbation

$$\psi' = \psi_0 \sin(kx + \phi)\cos ly \quad , \tag{150}$$

then Eq. (148) has the steady-state solution (i.e. dropping the partial time derivative) [exercise!]

$$\psi_0 = f_0 h_0 / [H(K^2 - K_s^2)] \quad . \tag{151}$$

The streamfunction is either exactly in phase (ridges over the mountains) or exactly out of phase (troughs over the mountains) with the topography depending on the sign $K^2 - K_s^2$. For long waves, ($K < K_s$), the topographic vorticity source in Eq. (148) is primarily balanced by meridional advection of planetary vorticity (the β effect). For short waves ($K > K_s$) the source is balanced primarily by the zonal advection of relative vorticity.

The topographic wave solution (151) has the unrealistic characteristic that when the wave number exactly equals the critical wave number K_s the amplitude goes to infinity. This is the resonant response case when the wave number reaches the stationary wave number of free Rossby waves.

Fig. 13 gives another example for a stationary Rossby wave, caused by ENSO forcing (discuss Eq. 109).

5.3 Turning Latitude

In reality the theory applied here with a constant β and u is a little to over-simplified, and a more correct treatment would make use of the dynamics in spherical coordinates (as in e.g. Hoskins and Karoly, 1981). However, we can derive some properties for an initially north-eastward propagating stationary Rossby wave here knowing that β slowly deceases in the meridional direction. Let us conisder the stationary Rossby wave 153

$$k^2 + l^2 = \beta / \overline{u} \quad . \tag{152}$$

Let's assume a wave generated by ENSO in the tropics moves north-eastward, and that its zonal wave number is a constant. If we further take into account that β decreases to the north with the cosine of the latitude, then the meridional wave number l must decrease until it becomes 0. From this point the wave turns southward again. The latitude in which this occurs is called *turning latitude*, and it is an important property of stationary Rossby waves generated in the tropics. Try to identify the turning latitude in Fig. 13.

We can go a step further, and let also the mean wind u depend on latitude, in which case Eq. 152 has an additional terms:

$$k^2 + l^2 = \frac{\left(\beta - \frac{d^2u}{dy^2}\right)}{\overline{u}} \quad . \tag{153}$$

There are some metric terms missing in this equation, but this expression gives a hint why a strong jet can modify stationary Rossby waves (the full correct expression can be found in Hoskins and Karoly, 1981). Strong jets are therefore also able to modify the turning latitude and other properties of stationary Rossby waves. Fig. 14 shows two examples of stationary wave number distribution in the meridional direction versus the zonal wavenumber for regions with a strong jet (South Asian/Western Pacific region; solid line) and one with a weaker jet (Eastern Pacific region; see Fig. 15). North of a strong jet the turning latitude is reduced, and we get an effect called *waveguide*, e.g. wave numbers 5 and 6 are essentially trapped in the region between 25° and 35°N. How do you determine the turning latitude in this graph?

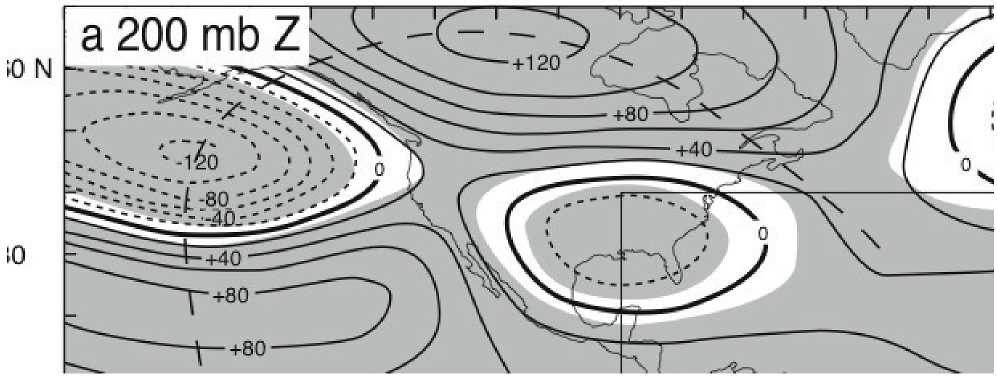

Figure 13: Stationary Rossby wave induced by ENSO.

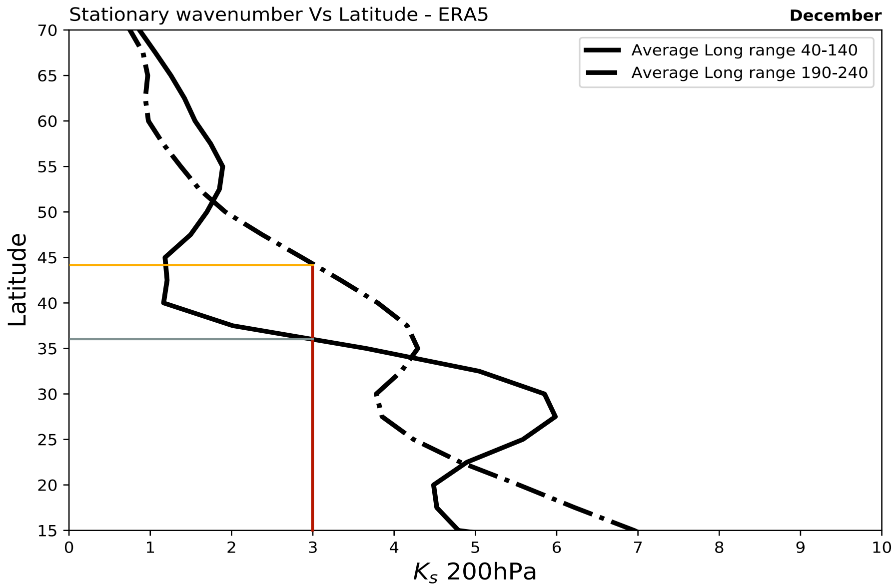

Figure 14: Meridional profile of stationary wave number (K_s). From Master thesis of Alessandro Raganato.

Exercises

1. Derive the group velocities for Rossby waves (145) and (146) and show that for stationary Rossby waves fulfilling Eq. (153), the c_{gx} component is always positive.

2. Show that (151) is the solution of (148) with (149).

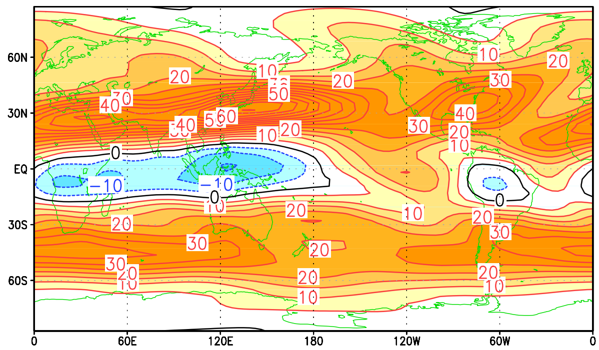

Figure 15: Northern winter (DJF) 200 hPa zonal wind. Units are m/s.